

A fuzzy method for solving unbalanced assignment problems with interval valued coefficients

K.L. KAGADE AND V.H. BAJAJ

Accepted : February, 2010

ABSTRACT

Most of the times in real life situations, it is difficult to balance between jobs and machines; therefore we face unbalanced assignment problems. The method suggested in the literature is based on the assumption to assign some of the jobs to dummy or fictitious machines. Those jobs which are executed on dummy machines may be ignored later. This paper concentrates on the unbalanced assignment problem where costs are not deterministic numbers but imprecise ones. The elements of the cost matrix of the unbalanced assignment problem are fuzzy intervals with linear membership function. We suggested a modified method for solving the fuzzy unbalanced assignment problems. The method is capable of assigning all the jobs to machine optimally. The method is presented in an algorithmic form and fuzzy programming technique is also used to develop this algorithm. Finally, numerical example has been given to illustrate the solution procedure.

Key words : Unbalanced assignment problem, Interval number, Fuzzy membership function, Fuzzy programming technique

The assignment problem is a special case of the L.P.P. It has been discussed with the situation in which jobs are to be assigned to a machine for execution. The linear programming formulation of the assignment problem and systematic method of solution was given by Hungarian mathematician D. Konig (2000). When we deal with real life situation, it becomes quite difficult to ensure that jobs are exactly equal to machines means problem goes to the unbalanced assignment problem. For solving such type of unbalanced assignment problem Kumar (2006) proposed a new modified method. But in realistic situation time / cost is imprecise, vague and flexible in nature *i.e.* the elements of the effective matrix should be imprecise number instead of fixed real numbers as because time / cost for doing a job by a facility (machine / person) might vary due to different reasons and their values are varied within some ranges (intervals). Alefeld and Herzberger (1983), and Moore (1979) gives complete details of interval arithmetic and operations on intervals. Therefore fuzzy programming approach is more appropriate for expressing real situation. In fuzzy decision making process, first Bellman and Zadeh (1970) introduced fuzzy set theory and Zimmerman (1985) expressed fuzzy set theory briefly.

Thus the need arises to solve the unbalanced assignment problem using fuzzy set theory in such a way that total assignment cost may be optimized along with the other constraints. The mathematical formulation of the interval assignment problem is as follows:

Interval assignment problem:

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n [c_{Lij}, c_{Rij}] x_{ij} \quad (1)$$

Subject to

$$\sum_{j=1}^n x_{ij} = 1, \quad i=1,2,\dots,m. \quad (2)$$

$$\sum_{i=1}^m x_{ij} = 1, \quad j=1,2,\dots,n. \quad (3)$$

$$x_{ij} = \begin{cases} 1, & \text{if } i \text{ assigned to } j \\ 0, & \text{if } i \text{ does not assigned to } j \end{cases} \quad (4)$$

where $[C_{Lij}, C_{Rij}]$ is an interval representing the uncertain cost for the assignment problem.

Setting $M = 1, 2, \dots, m$, $N = 1, 2, \dots, n$ and $J = \{(i, j) / i \in M, j \in N\}$, the problem may be restated as

$$\text{Minimize } \left\{ Z = \sum_{(i,j) \in J} c_{ij} x_{ij} \left| \begin{array}{l} \sum_j x_{ij} = 1 \\ \sum_i x_{ij} = 1 \\ x_{ij} = 0, 1; \forall i, j. \end{array} \right. \right\} \quad (5)$$

Correspondence to:

K.L. KAGADE, Department of Statistics, Dr. Babasaheb Ambedkar Marathwada University, AURANGABAD (M.S.) INDIA

Authors' affiliations:

V.H. BAJAJ, Department of Statistics, Dr. Babasaheb Ambedkar Marathwada University, AURANGABAD (M.S.) INDIA